

Pathway2Careers Algebra I









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About NS4ed

NS4ed is an educational research and development company dedicated to negotiating services for schools, educators, and private institutions. As a rising leader in professional guidance in education, NS4ed partners with state and local entities to provide research, policy and practice deliverables that yield high value and actionable results. NS4ed offers a unique perspective on gathering data and understanding the policy effect and implications for developing models that yield mission critical change in a community, district, or organization.



About Pathway2Careers

As NS4ed's flagship program, Pathway2Careers (P2C) is a career readiness solution that support schools in their efforts to prepare students for high-value careers in their communities. P2C delivers access to labor market data to encourage the use of data-informed practices in career education through a number of services, including labor market data tools, career-focused curricula, Perkins V support, and more. As a comprehensive career readiness solution, P2C serves administrators, faculty, students, and parents in making informed decisions that promote employment success.

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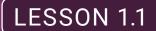
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7. Sequences Lesson 7.1	Comparing Exponential Functions S Lesson Topic Understanding Sequences	E.A.1.a A1.F.IF.C.9, A1.F.IF.C.9.a, A1.F.LE.A .1, A1.F.LE.A.1.a TN State Standard A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.BF.A.1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F.LE.A.1.a, A1.F.LE. A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2, A1.F.IF.A.2, A1. 1, A1.F.BF.A.1.a, A1.F.IE.A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C	2, 4, 7 Mathematical Practices 1, 4, 8	Occupation Biological Science Teachers, Postsecondary
7. Sequences Lesson 7.1 Lesson 7.2	Comparing Exponential Functions Lesson Topic Understanding Sequences Explicitly-Defined Arithmetic Sequences	E.A.1.a A1.F.IF.C.9, A1.F.IF.C.9.a, A1.F.LE.A .1, A1.F.LE.A.1.a TN State Standard A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.BF.A.1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F.LE.A.1.a, A1.F.LE. A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.IF.A.2 1, A1.F.BF.A.1.a, A1.F.LE.A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2	2, 4, 7 Mathematical Practices 1, 4, 8	Occupation Biological Science Teachers, Postsecondary Multiple
7. Sequences Lesson 7.1	Comparing Exponential Functions S Lesson Topic Understanding Sequences	E.A.1.a A1.F.IF.C.9, A1.F.IF.C.9.a, A1.F.LE.A .1, A1.F.LE.A.1.a TN State Standard A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.BF.A.1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F.LE.A.1.a, A1.F.LE. A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A. 1, A1.F.BF.A.1.a, A1.F.LE.A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A.	2, 4, 7 Mathematical Practices 1, 4, 8	Occupation Biological Science Teachers, Postsecondary
7. Sequences Lesson 7.1 Lesson 7.2	Comparing Exponential Functions Lesson Topic Understanding Sequences Explicitly-Defined Arithmetic Sequences	E.A.1.a A1.F.IF.C.9, A1.F.IF.C.9.a, A1.F.LE.A .1, A1.F.LE.A.1.a TN State Standard A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.BF.A.1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F.LE.A.1.a, A1.F.LE. A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A. 1, A1.F.BF.A.1.a, A1.F.LE.A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A. 1, A1.F.BF.A.1.a, A1.F.IF.A.2, A1.	2, 4, 7 Mathematical Practices 1, 4, 8	Occupation Biological Science Teachers, Postsecondary Multiple
7. Sequences Lesson 7.1 Lesson 7.2	Comparing Exponential Functions Lesson Topic Understanding Sequences Explicitly-Defined Arithmetic Sequences	E.A.1.a A1.F.IF.C.9, A1.F.IF.C.9.a, A1.F.LE.A .1, A1.F.LE.A.1.a TN State Standard A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.BF.A.1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F.LE.A.1.a, A1.F.LE. A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A. 1, A1.F.BF.A.1.a, A1.F.LE.A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A.	2, 4, 7 Mathematical Practices 1, 4, 8	Occupation Biological Science Teachers, Postsecondary Multiple Multiple
7. Sequences Lesson 7.1 Lesson 7.2	Comparing Exponential Functions Lesson Topic Understanding Sequences Explicitly-Defined Arithmetic Sequences	E.A.1.a A1.F.IF.C.9, A1.F.IF.C.9.a, A1.F.LE.A .1, A1.F.LE.A.1.a TN State Standard A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.BF.A.1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F.LE.A.1.a, A1.F.LE. A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A. 1, A1.F.BF.A.1.a, A1.F.LE.A.2 A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.1, A1.F.IF.A.2, A1. F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A. 1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F. LE.A.1.a, A1.F.LE.A.2	2, 4, 7 Mathematical Practices 1, 4, 8	Occupation Biological Science Teachers, Postsecondary Multiple

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		A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1.		
Lesson 7.5	Explicitly-Defined Geometric Sequences	F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A.	2, 7, 8	Multiple
		1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F.	_,,,,	
		LE.A.1.c, A1.F.LE.A.2		
		A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C		
		ED.A.3, A1.F.IF.A.1, A1.F.IF.A.2, A1.		
Lesson 7.6	Recursively-Defined Geometric Sequences	F.IF.A.2.a, A1.F.IF.A.2.b, A1.F.BF.A.	2, 7, 8	Multiple
		1, A1.F.BF.A.1.a, A1.F.LE.A.1, A1.F.		
		LE.A.1.a, A1.F.LE.A.1.c, A1.F.LE.A.2		
1 7 7		A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C		
Lesson 7.7	Applying Geometric Sequences	ED.A.3, A1.F.BF.A.1, A1.F.BF.A.1.a, A1.F.LE.A.2	1, 2, 4	Actuaries
		A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C		
Lesson 7.8	Applying Recursively-Defined Sequences	ED.A.3, A1.F.BF.A.1, A1.F.BF.A.1.a,	6, 7, 8	Sociologists
		A1.F.BF.B.2, A1.F.LE.A.2		
8. Polynomi	als and Factoring			
	Lesson Topic	TN State Standard	Mathematical Practices	Occupation
Lesson 8.1	Modeling with Polynomials	A1.A.SSE.A.1, A1.A.SSE.A.1.a, A1.A.	1, 2, 4	Astronomers
		SSE.A.1.b	-, -, +	
Lesson 8.2	Adding and Subtracting Polynomials	A1.A.SSE.A.1, A1.A.SSE.A.1.a, A1.A.	1, 7, 8	Multiple
		SSE.A.1.b, A1.A.APR.A.1	_, , , 0	
Lesson 8.3	Multiplying Monomial and Binomials	A1.A.SSE.A.1, A1.A.SSE.A.1.a, A1.A.	1, 2, 7	Multiple
		SSE.A.1.b, A1.A.APR.A.1	. ,	•
Lesson 8.4	Multiplying Polynomials	A1.A.SSE.A.1, A1.A.SSE.A.1.a, A1.A.	1, 2, 7	Multiple
		SSE.A.1.b, A1.A.APR.A.1		
Lesson 8.5	Special Products of Polynomials	A1.A.SSE.A.1, A1.A.SSE.A.1.a, A1.A. SSE.A.1.b, A1.A.APR.A.1	1, 7, 8	Multiple
1 a a a a a a a a	Out and the could be be a could be	A1.A.APR.A.1, A1.A.REI.B.3, A1.A.R	1.2.0	Operations Research
Lesson 8.6	Operations with Polynomials	EI.B.3.a	1, 2, 8	Analyst
Lesson 8.7	Factoring Using the GCF	A1.A.REI.B.3, A1.A.REI.B.3.a	1, 2, 7	Multiple
Lesson 8.8	Factoring the Difference of Squares	A1.A.APR.A.1, A1.A.REI.B.3, A1.A.R	1, 2, 7	Multiple
		EI.B.3.a	_,_,	
Lesson 8.9	Factoring Using Grouping	A1.A.APR.A.1, A1.A.REI.B.3, A1.A.R	2, 7, 8	Multiple
1 0 10		EI.B.3.a		
Lesson 8.10	Factoring Quadratic Expressions in Form x ² + Bx + C	A1.A.REI.B.3, A1.A.REI.B.3.a	1, 2, 7	Multiple
Lesson 8.11	Factoring Quadratic Expressions in Form Ax ² + Bx + C	A1.A.REI.B.3, A1.A.REI.B.3.a	1, 2, 7	Multiple
		A1.A.REI.A.1, A1.A.REI.B.2, A1.A.R		
Lesson 8.12	Solving Polynomial Equations	EI.B.2.a, A1.A.REI.B.3, A1.A.REI.B.3	1, 7, 8	Multiple
		.a, A1.F.IF.C.8, A1.F.IF.C.8.a		
9. Quadrati	c Functions and Equations			
	Lesson Topic	TN State Standard	Mathematical Practices	Occupation
		A1.A.CED.A.2, A1.A.REI.D.5, A1.F.IF		
Lesson 9.1	Graphing Quadratic Functions in Standard Form	.B.4, A1.F.IF.C.7, A1.F.LE.B.3	2, 4, 7	Multiple
		A1.A.CED.A.2, A1.A.REI.B.3, A1.A.REI.B.3.a		
Lesson 9.2	Graphing Quadratic Functions in Vertex Form and Intercept Form	, A1.A.REI.D.5, A1.F.IF.B.4, A1.F.IF.B.5, A1.	1247	Multiple
EC33011 3.2	S. Apriling Quadri acte i unctions in vertex i of ill and intercept rollin	F.IF.C.7, A1.F.IF.C.8, A1.F.IF.C.8.a, A1.F.LE.	1, 2, 4, 7	water
		B.3		
Logger 0.2	Applying the Venter Form of Orad - 1'- Free 1'-	A1.A.CED.A.2, A1.A.REI.B.3, A1.A.R	1.2.1	Atmospheric and Space
Lesson 9.3	Applying the Vertex Form of Quadratic Functions	EI.B.3.a, A1.F.IF.C.8, A1.F.IF.C.8.a	1, 2, 4	Scientists
		A1.A.CED.A.2, A1.A.REI.B.3, A1.A.R		
Lesson 9.4	Applying Graphs of Quadratic Functions	EI.B.3.a, A1.F.IF.C.8, A1.F.IF.C.8.a	4, 6, 7	Aerospace Engineers
		Endida, Aliminico, Aliminico.a		
Lesson 9.5	Solving Quadratic Equations by Graphing and Taking the Square Root	A1.A.CED.A.2, A1.A.REI.B.3, A1.A.R	1, 2, 5, 7	Multiple
203011 3.3	25 Quadratic Equations by Graphing and Taking the Square ROOL	EI.B.3.a, A1.A.REI.D.6	±, ±, 3, 1	manpic
		A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C		
Lesson 9.6	Solve Quadratic Equations by Factoring	ED.A.3, A1.A.REI.B.3, A1.A.REI.B.3. a	1, 2, 4, 8	Multiple
		A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C		
Lesson 9.7	Solve Quadratic Equations by Completing the Square	ED.A.3, A1.A.REI.B.3, A1.A.REI.B.3.	1, 2, 7	Multiple
	Annual and Education of combinering are added to	a, A1.F.IF.C.8, A1.F.IF.C.8.a	-, -, ,	
		A1.A.REI.B.3, A1.A.REI.B.3.a, A1.F.I		
Lesson 9.8	Solve Quadratic Equations by Quadratic Formula	F.C.8, A1.F.IF.C.8.a	1, 2, 4, 5	Multiple

Lesson 11.10 Lesson 11.11	Inverses of Radical Functions Inverse of Quadratic Functions	A1.F.IF.B.4, A1.F.IF.B.5 A1.F.IF.B.4, A1.F.IF.B.5	1, 2, 4 1, 2, 4	Multiple Multiple
Lesson 11.9	Inverses of Linear Functions	A1.F.IF.B.4, A1.F.LE.B.3	1, 2, 4	Multiple
Lesson 11.8	Inverses of Functions	A1.F.IF.B.4, A1.F.LE.B.3	1, 2, 4	Multiple
Lesson 11.7	Solving Radical Equations	A1.A.REI.D.6	1, 2, 5, 7	Multiple
Lesson 11.6	Describing and Graphing Cube Root Functions	A1.A.CED.A.2, A1.A.CED.A.3, A1.A.R EI.D.5, A1.F.IF.B.4, A1.F.IF.B.5, A1. F.IF.C.7, A1.F.BF.B.2, A1.F.LE.B.3	1, 2, 7	Multiple
Lesson 11.5	Applying Graphs of Square Root Functions	A1.A.CED.A.2, A1.A.REI.D.5, A1.F.IF .B.4, A1.F.IF.B.5, A1.F.IF.C.7, A1.F.L E.B.3	1, 2, 4	Mechanical Engineers
Lesson 11.4	Applying Square Root Functions	A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3	1, 2, 4	Registered Nurses
Lesson 11.3	Writing Square Root Functions	A1.A.CED.A.2, A1.A.CED.A.3, A1.A.R EI.D.5, A1.F.IF.B.4	1, 2, 4	Radiologic Technologists
Lesson 11.2	Describing and Graphing Square Root Functions	A1.A.CED.A.2, A1.A.CED.A.3, A1.A.R EI.D.5, A1.F.IF.B.4, A1.F.IF.B.5, A1. F.IF.C.7, A1.F.BF.B.2, A1.F.LE.B.3	1, 6, 7	Multiple
Lesson 11.1	Radical Expressions	A1.N.Q.A.1.c	Practices 2, 6, 7	Multiple
- 11. Kadical	Expressions and Inverse Functions Lesson Topic	TN State Standard	Mathematical	Occupation
	Applying Comparisons of Linear, Exponential, and Quadratic Models	A1.F.IF.B.6, A1.F.IF.C.9, A1.F.IF.C.9 .a, A1.F.IF.C.9.b	1, 2, 4	Appraisers and Assessors of Real Estate
Lesson 10.10	Comparing Linear, Exponential, and Quadratic Models	A1.F.IF.B.6, A1.F.IF.C.9, A1.F.IF.C.9 .a, A1.F.IF.C.9.b, A1.F.IE.A.1, A1.F.L E.A.1.a, A1.F.IE.A.1.b, A1.F.IE.A.1.c	6, 7, 8	Multiple
Lesson 10.9	Operations on Functions	A1.A.CED.A.2, A1.A.REI.D.5, A1.F.IF .B.4, A1.F.BF.B.2	4, 7, 8	Film and Video Editors
Lesson 10.8	Reflection of Graphs of Functions	A1.A.CED.A.2, A1.A.REI.D.5, A1.F.IF .B.4, A1.F.BF.B.2	1, 6, 7, 8	Multiple
Lesson 10.7	Stretches and Shrinks of Graphs of Functions	B.2, A1.F.LE.A.2 A1.A.CED.A.2, A1.A.REI.D.5, A1.F.IF .B.4, A1.F.BF.B.2	1, 2, 7	Multiple
Lesson 10.6	Translations of Graphs of Functions	A1.A.CED.A.1, A1.A.CED.A.2, A1.A.C ED.A.3, A1.A.REI.D.5, A1.F.IF.B.4, A 1.F.BF.A.1, A1.F.BF.A.1.a, A1.F.BF.	1, 2, 7	Multiple
Lesson 10.5	Applying Piecewise-Defined Functions	F.LE.B.3 A1.A.CED.A.2, A1.A.REI.D.5, A1.F.IF .B.4, A1.F.IF.B.5, A1.F.IF.C.7, A1.F.L E.B.3	1, 2, 4	Tax Preparers
Lesson 10.4	Graphing Piecewise-Defined Functions	E.B.3 A1.A.CED.A.2, A1.A.CED.A.3, A1.A.R EI.D.5, A1.F.IF.B.4, A1.F.IF.C.7, A1.	1, 2, 6, 7	Multiple
Lesson 10.3	Applying Step Functions	F.LE.B.3 A1.A.CED.A.2, A1.A.REI.D.5, A1.F.IF .B.4, A1.F.IF.B.5, A1.F.IF.C.7, A1.F.L	1, 2, 4	Cargo and Freight Agents
Lesson 10.2	Graphing Step Functions	F.LE.B.3 A1.A.CED.A.2, A1.A.CED.A.3, A1.A.R EI.D.5, A1.F.IF.B.4, A1.F.IF.C.7, A1.	1, 2, 7	Multiple
Lesson 10.1	Graphing Absolute Value Functions	A1.A.CED.A.2, A1.A.CED.A.3, A1.A.R EI.D.5, A1.F.IF.B.4, A1.F.IF.C.7, A1.	1, 2, 7	Multiple
	Lesson Topic	TN State Standard	Mathematical Practices	Occupation
10. Graphin	g and Modeling with Functions			
Lesson 9.13	Applying Linear-Quadratic Systems Applying Linear-Quadratic Systems	A1.A.REI.D.6	1, 2, 4	Economists
Lesson 9.11 Lesson 9.12	Solving Linear-Quadratic Systems Graphically Solving Linear-Quadratic Systems Algebraically	A1.A.REI.B.3, A1.A.REI.B.3.b, A1.A. REI.D.6 A1.A.REI.D.6	1, 2, 4, 5 2, 6, 7	Multiple Multiple
Lesson 9.10	Comparing Quadratic Functions	A1.A.CED.A.2, A1.A.REI.D.5, A1.F.IF .B.4, A1.F.IF.C.7, A1.F.IF.C.9, A1.F.I F.C.9.a, A1.F.LE.B.3	1, 2, 4	Industrial Production Managers
Lesson 9.9	Using Quadratic Equations to Solve Problems	A1.A.REI.B.3, A1.A.REI.B.3.a, A1.F.I F.C.8, A1.F.IF.C.8.a	1, 4, 6	Physicists

Lesson 11.12	Applying Inverse Functions	A1.F.IF.B.4, A1.F.IF.B.5	1, 2, 6	Wind Turbine Service Technicians	
12. Statistic	s				
	Lesson Topic	TN State Standard	Mathematical Practices	Occupation	
Lesson 12.1	Measures of Center	A1.S.ID.A.1, A1.S.ID.A.3	1, 3, 5	Multiple	
Lesson 12.2	Measures of Spread	A1.S.ID.A.3	1, 3, 5	Multiple	
Lesson 12.3	Applying Measures of Center and Spread	A1.S.ID.A.1, A1.S.ID.A.3	1, 2, 4	Statisticians	
Lesson 12.4	Representing Data with Box Plots	A1.S.ID.A.3	1, 2, 7	Multiple	
Lesson 12.5	Distributions of Data	A1.S.ID.A.2, A1.S.ID.A.3	2, 4, 7	Multiple	
Lesson 12.6	Applying Box Plots	A1.S.ID.A.3	2, 4, 5	Computer and Information Systems Managers	
Lesson 12.7	Representing Data with Histograms	A1.S.ID.A.2, A1.S.ID.A.3	3, 6, 7	Multiple	
Lesson 12.8	Applying Histograms	A1.S.ID.A.2, A1.S.ID.A.3	2, 4, 5	Financial Examiners	
Lesson 12.9	Analyzing Data	A1.S.ID.A.2, A1.S.ID.A.3	2, 3, 4	Market Research Analysts and Marketing Specialists	
Lesson 12.10	Two-Way Frequency Tables	A1.S.ID.A.2	1, 2, 7	Multiple	
Lesson 12.11	Applying Two-Way Frequency Tables	A1.S.ID.A.2	1, 3, 5	Social Science Research Assistants	



Real Numbers





Encourage your students to learn more about these occupations in the Pathway2Careers Career Library.

Lesson Objective

In this lesson, you will understand properties of real numbers.

- You will understand that rational numbers and irrational numbers make up real numbers.
- You will understand that the sums and products of rational numbers are rational numbers.

Vocabulary

- rational numbers
- real numbers
- irrational numbers
- closure

Teaching Support

Number Sense Essentials

Teaching Strategy Students should become familiar with the diagram and understand that if a real number is not rational, it must be irrational. Explain that in later courses, they will learn about other numbers that are not real numbers.

Classifying Real Numbers

Avoid Common Errors Make sure that students understand that having the square root symbol does not mean that a number is irrational. For example, $\sqrt{4}$ simplifies to 2, a rational number.

Example 1 Classifying Real Numbers

Cooperative Learning Have students work in small groups to write irrational and rational numbers. Then have groups exchange their numbers to check each other's understanding.

Example 2 Determining an Irrational Number

Teaching Strategy Note that this argument can be used for any prime number under the square root symbol.

Build Your Skills: Classifying Real Numbers

Answers

- 1. irrational; It cannot be written as a ratio of integers.
- 2. rational; It can be written as the ratio $\frac{2}{3}$.
- 3. rational; It can be written as the ratio $-\frac{5}{3}$.
- **4.** rational; It can be written as the ratio $\frac{16}{5}$.
- **5.** Assume $\sqrt{5}$ is a rational number written in simplest form $\frac{p}{q}$, where p and q are integers. By squaring, $\frac{p^2}{q^2} = 5$, so p^2 is divisible by 5, which means 5 is also a factor of p because 5 is a prime number. So, p^2 is divisible by 25 and that implies 5 is factor q^2 and a factor of q. This means $\frac{p}{q}$ is not in simplest form, which contradicts the assumption, so $\sqrt{5}$ must be an irrational number.

Number Sense Essentials

Teaching Strategy Closure is an important concept for students to understand as they will later explore the closure of polynomials under addition, subtraction, and multiplication.

Extension Closure can also be determined for subsets. Ask students whether even integers form a closed set under addition, subtraction, and multiplication.

Exploring Properties of Real Numbers

Example 3 Identifying Rational and Irrational Numbers

Teaching Strategy Have students label each number as irrational or rational to help them recognize which property is being applied. For example, for Example 3a, students can write: rational + irrational. This is an example of the third property, so the sum is irrational.

Example 4 Adding a Rational Number and an Irrational Number

Teaching Strategy Point out that since a real number is either rational or irrational, if a number is not rational, it must be irrational.

Teaching Strategy Students can use a similar argument to show that a product of a rational and an irrational number is irrational. Discuss how students can use operations to show that the sum and product of two rational numbers are rational.

Build Your Skills: Exploring Properties of Real Numbers

Answers

- **6.** rational; It is the product of two rational numbers.
- 7. irrational; It is the sum of an irrational number and a rational number.
- 8. irrational; It can be written as the product of an irrational number and a rational number.
- **9.** rational; It is the difference of two rational numbers.
- **10.** Assume $\sqrt{5} + \left(\frac{3}{4}\right)$, or $\sqrt{5} \frac{3}{4}$, is a rational number written in simplest form $\frac{p}{q}$, where p and q are integers. Add $\frac{3}{4}$ to each side of the equation $\sqrt{5} \frac{3}{4} = \frac{p}{q}$, so that means $\sqrt{5} = \frac{p}{q} + \frac{3}{4} = \frac{4p + 3q}{4q}$, which is a rational number. This is a contradiction that $\sqrt{5}$ is irrational, so the sum must be irrational.
- 11. Assume 2.3π is a rational number written in simplest form $\frac{p}{q}$, where p and q are integers. Multiply each side of the equation $2.3\pi = \frac{p}{q}$ by $\frac{1}{2.3}$, so $\pi = \frac{p}{2.3q}$. Since p, 2.3, and q are rational, 2.3q and $\frac{p}{2.3q}$ are rational. This is a contradiction that π is irrational, so the product must be irrational.

Career Preparation: Practice

Avoid Common Errors In Exercise 5, students may think the number is irrational. Point out that the decimal has a definite end at 9, so it can be written as the ratio $\frac{314,159}{100,000}$.

Teaching Strategy For Exercises 10, students should refer to the argument used in Example 2.

Answers

- 1. rational; It can be written as the ratio $\frac{3}{7}$.
- 2. rational; It is written as the ratio $\frac{5}{8}$
- 3. irrational; It cannot be written as a ratio of integers.
- 4. irrational; It cannot be written as a ratio of integers.
- **5.** rational; It can be written as the ratio $\frac{314,159}{100,000}$.
- **6.** rational; It can be written as a ratio $\frac{-8}{11}$.
- 7. rational; It can be written as the ratio $\frac{-11}{12}$.
- 8. irrational; It cannot be written as a ratio of integers.
- 9. irrational; It cannot be written as a ratio of integers.
- **10.** Assume that $\sqrt{7}$ is a rational number, so it can be written as the ratio of two integers p and q. $\frac{p}{q} = \sqrt{7}$. Assume that the integers p and q are both positive and chosen so that $\frac{p}{q}$ is in simplest form. $\frac{p^2}{q^2} = 7$, so $7q^2 = p^2$. So, p^2 is divisible by 7 and p = 7m for a positive integer m. $7q^2 = (7m)^2 = 7^2m^2$. $q^2 = 7m^2$. So, q^2 is divisible by 7. q = 7n, for a positive integer p. Because 7 is factor p and 7 is a factor of p, $\frac{p}{q}$ is not in simplest form. This is a contradiction. So, $\sqrt{7}$ is irrational.
- **11.** $\sqrt{4} = 2 = \frac{2}{1}$. So, $\sqrt{4}$ is rational.
- **12.** Because π is an irrational number and $(2)^2$ is a rational number, the product is irrational.
- **13.** Because both 6 and $\frac{3}{4}$ are rational numbers, the sum is rational.
- **14.** Because both $\frac{3}{4}$ and $\frac{9}{10}$ are rational numbers, the difference is rational.
- **15.** Because both $0.\overline{63} = \frac{7}{11}$ and $\frac{1}{3}$ are rational numbers, the sum is rational.
- **16.** Because $\sqrt{37}$ is an irrational number and 1 is a rational number, the sum is irrational.
- 17. Because -6.5749 is a rational number and $\sqrt{15}$ is an irrational number, the sum is irrational.

- **18.** Because $\left(\sqrt{3}\right)^2 = 3 = \frac{3}{1}$, $\left(\sqrt{3}\right)^2$ is rational.
- **19.** Because $\frac{1}{5}$ is a rational number and $3.\overline{5} = 3\frac{5}{9}$ is a rational number, the product is rational.
- **20.** $-(\sqrt{5^3}) = -\sqrt{125}$ is irrational because 125 is not a perfect square.
- **21.** $\left(\frac{7}{16}\right)\left(\frac{2}{5}\right) = \frac{7}{40}$ is a rational number.
- **22.** $\frac{3}{4} + \frac{4}{5} = \frac{31}{20}$ is a rational number.
- **23.** Possible answer: $\sqrt{2}$; If $s = \sqrt{2}$, then $d = \sqrt{2} \cdot \sqrt{2} = \left(\sqrt{2}\right)^2 = 2 = \frac{2}{1}$, so d is a rational number.
- **24.** Possible answer: 5; If $\ell = 5$ then $s = 2\sqrt{5 \cdot 5} = 2\sqrt{25} = 2(5) = 10 = \frac{10}{1}$, so s is a rational number.

Career Preparation: Check

Test-Taking Tip Students should pay attention for perfect square numbers under the square root sign. These numbers are rational numbers.

Answers

1. D

2. B

3. B

4. B

5. C

6. A

- **7.** a, b, e, f
- **8.** b, c, e

- 9. True, False, False, True, False
- **10.** B

11. D

LESSON 1.1

Real Numbers



CAREER PREPARATION: Essential Number Sense Skills



Did you know?

Physicists use different types of numbers to examine and understand physical phenomena.

Consider this situation...

A physicist makes a calculation to determine how long it will take a ball to fall 32 feet. Her calculation shows that the time it takes for the ball to fall is $\sqrt{2}$, or about 1.4142, seconds. Explain why $\sqrt{2}$ is not a rational number.



To determine the time it takes, the physicist solves the equation $0 = -16t^2 + 32$. In solving this equation, the physicist gets $t = \sqrt{2}$.

A rational number is any number that can be written as ratio of two integers. It can be shown that $\sqrt{2}$ is not a ratio of two integers.

Let's find out... why $\sqrt{2}$ is not a rational number so you can classify rational and irrational numbers on your own in the Career Preparation Exercises.

Lesson Objective

In this lesson, you will understand properties of real numbers.

- You will understand that rational numbers and irrational numbers make up real numbers.
- You will understand that the sums and products of rational numbers are rational numbers.

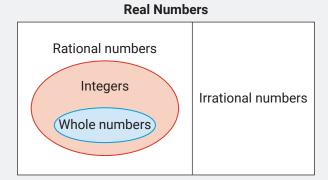


Number Sense Essentials

Recall that **rational numbers** are all numbers that can be written in the form of a ratio $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Rational numbers are part of the *real numbers*. **Real numbers** are all numbers that can be represented on a number line. Any real number that is not a rational number is an *irrational number*. **Irrational numbers** are all real numbers that cannot be written as a ratio of integers. The decimal form of an irrational is both non-terminating and non-repeating.

For example, the numbers $\sqrt{2}$ and π are irrational numbers. Their decimal approximations 1.21421... and 3.14159... are non-terminating and non-repeating.

The diagram shows how real numbers can be classified as whole numbers, integers, rational numbers, and irrational numbers.



Classifying Real Numbers

Example 1 Classifying Real Numbers

Determine whether the real number is rational or irrational. Explain your answer.

a. 5

b. $\sqrt{11}$

c. $-\frac{21}{4}$

d. 2.4

Solution

- **a.** The number 5 can be written as the ratio $\frac{5}{1}$, so it is a rational number.
- **b.** The number $\sqrt{11}$ cannot be written as a ratio of integers, so it is an irrational number.
- **c.** The number $-\frac{21}{4}$ can be written as the ratio $\frac{-21}{4}$, so it is a rational number.
- **d.** The number 2.4 can be written as the ratio $\frac{12}{5}$, so it is a rational number.

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Example 2 Determining an Irrational Number

Show why $\sqrt{2}$ is not a rational number.

Solution

Assume that $\sqrt{2}$ is a rational number, so it can be written as the ratio of two integers p and q.

$$\frac{p}{q} = \sqrt{2}$$

You can also assume that the integers p and q are both positive and chosen so that $\frac{p}{q}$ is in simplest form. Then, square $\frac{p}{q}$ and $\sqrt{2}$, so you get the following.

$$\frac{p^2}{q^2}=2$$

So, p^2 is divisible by 2, which means 2 must be a factor of p because 2 is a prime number. This means p^2 is divisible by 4. This implies that 2 must be a factor q^2 since the quotient of p^2 and q^2 is 2. This means 2 is a factor of q.

Because 2 is factor q and 2 is a factor of p, this tells you that $\frac{p}{q}$ is not in simplest form. This is a contradiction. In math, if you make an assumption that leads to a contradiction, the assumption cannot be true. Therefore, the assumption that $\sqrt{2}$ is a rational number is not true, so $\sqrt{2}$ must be an irrational number.

Build Your Skills: Classifying Real Numbers

Determine whether the real number is rational or irrational. Explain your answer.

1.
$$-\sqrt{3}$$

2.
$$0.\overline{6}$$

3.
$$-\sqrt{\frac{25}{9}}$$

5. Show why
$$\sqrt{5}$$
 is not a rational number.

Did you know?

Mechanical drafters make designs that involve calculations with irrational numbers such π and $\sqrt{2}$.



Number Sense Essentials

The set of real numbers is closed, or has closure, under addition, subtraction, and multiplication. **Closure** of a set of numbers for a given operation means that the operation of any two numbers in the set results in another number in the set. For example, the sum of two real numbers is another real number, so the set of real numbers is closed under addition. When accounting for closure of an operation, you must consider all numbers in the set. For example, the set of real numbers is not closed under division because dividing by zero does not result in a real number.

The set of rational numbers is also closed under addition, subtraction, and multiplication, and the set of nonzero rational numbers is closed under division. Irrational numbers are not closed under addition, subtraction, and multiplication.

The following properties are true:

- The sum and difference of two rational numbers are rational numbers.
- The product of two rational numbers is a rational number.
- The sum of an irrational number and a rational number is an irrational number.
- The product of an irrational number a rational number is an irrational number.

Exploring Properties of Real Numbers

Example 3 Identifying Rational and Irrational Numbers

Identify whether the number is rational or irrational. Explain your answer.

a.
$$1.3 + \pi$$

b.
$$0.75 - \frac{3}{14}$$
 c. $-5(4.\overline{3})$ **d.** $\frac{\sqrt{2}}{5}$

c.
$$-5(4.\overline{3})$$

d.
$$\frac{\sqrt{2}}{5}$$

Solution

- **a.** Because 1.3, which can be written as $\frac{8}{5}$, is a rational number and π is an irrational number, the sum is an irrational number.
- **b.** Because both 0.75, which can be written as $\frac{3}{4}$, and $\frac{3}{14}$ are rational numbers, the difference is a rational number.
- c. Because –5, which can be written as $\frac{-5}{1}$, and $4.\overline{3}$, which can be written as $\frac{13}{3}$, are rational numbers, the product is a rational number.
- **d.** The quotient $\frac{\sqrt{2}}{5}$ can be rewritten as the product $\frac{1}{5} \cdot \sqrt{2}$. Because $\frac{1}{5}$ is a rational number and $\sqrt{2}$ is an irrational number, the product is irrational.

Photo credit: Gorodenkoff/Shutterstock.com

Example 4 Adding a Rational Number and an Irrational Number

Show why the sum of the rational number 5 and the irrational number $\sqrt{11}$ is an irrational number.

Solution

Assume that $5 + \sqrt{11}$ is a rational number, so it can be written as the ratio of two integers p and q.

$$\frac{p}{q}=5+\sqrt{11}$$

Then, if you subtract 5 from $5 + \sqrt{11}$, it is the same as subtracting 5 from $\frac{p}{q}$. You can subtract 5 by writing it as a fraction with the common denominator q.

$$\frac{p}{q} - 5 = \frac{p}{q} - \frac{5q}{q} = \frac{p - 5q}{q}$$

The numerator p-5q and the denominator q are integers. This shows that $\sqrt{11}$ can be written as the ratio of two integers, which means $\sqrt{11}$ is a rational number, and that is a contradiction. The assumption that $5+\sqrt{11}$ is a rational number is not true, so $5+\sqrt{11}$ must be an irrational number.

Build Your Skills: Exploring Properties of Real Numbers

Identify whether the number is rational or irrational. Explain your answer.

6.
$$1.\overline{72} \left(\frac{3}{2} \right)$$

7.
$$-\sqrt{2} + 1.41$$

8.
$$\frac{\pi}{3}$$

9.
$$-3.14 - \frac{4}{7}$$

Show why the statement is true.

- **10.** The sum of the irrational number $\sqrt{3}$ and the rational number $-\frac{3}{4}$ is an irrational number.
- **11.** The product of the rational number 2.3 and the irrational number π is an irrational number.

Did you know?

Statisticians make calculations with data involving rational and irrational numbers to determine statistical measurements.



Career Preparation: Practice

Determine whether the real number is rational or irrational. Explain your answer.

2.
$$\frac{5}{8}$$

3.
$$\frac{\pi}{2}$$

4.
$$\sqrt{17}$$

6.
$$-\sqrt{\frac{64}{121}}$$

7.
$$-\frac{11}{12}$$

9.
$$\sqrt{\frac{2}{49}}$$

10. Show why $\sqrt{7}$ is not a rational number.

11. Show why $\sqrt{4}$ is a rational number.

Identify whether the number is rational or irrational. Explain your answer.

12.
$$\pi(2^2)$$

13.
$$6 + \frac{3}{4}$$

14.
$$\frac{3}{4} - \frac{9}{10}$$

15.
$$0.\overline{63} + \frac{1}{3}$$

16.
$$\sqrt{37} + 1$$

17.
$$-6.5749 + \sqrt{15}$$

18.
$$(\sqrt{3})^2$$

19.
$$\frac{1}{5}(3.\overline{5})$$

20.
$$-(\sqrt{5^3})$$

Show why the statement is true.

21. The product of the rational number $\frac{7}{16}$ and the rational number $\frac{2}{5}$ is a rational number.

22. The sum of the rational number $\frac{3}{4}$ and the rational number $\frac{4}{5}$ is a rational number.

Use It On the Job

23. Renata is a video game designer making an avatar move diagonally across a square room. The length of the diagonal d of a square of side s is $d = s\sqrt{2}$.



Determine a value of s for which d is a rational number. Explain your answer.

24. A police officer is using the formula $s = 2\sqrt{5\ell}$ to estimate the speed s of a car in miles per hour determined by the length ℓ in feet of the skid marks the car makes.



Determine a value of ℓ for which s is a rational number. Explain your answer.

Career Preparation: Check

- 1. Which number is rational?
 - A. $\sqrt{5}$
- C. $\pi\sqrt{5}$

- B. π
- D. $\sqrt{9}$

- 2. Which number is irrational?
 - **A.** $\frac{3}{5}$
- c. $\frac{7}{3}$
- B. $\frac{\sqrt{3}}{2}$
- **D.** $\sqrt{144}$

- **3.** Which number is *not* rational?
 - A. $\sqrt{4}$
- **c.** $\sqrt{12} \cdot \sqrt{3}$
- B. $\sqrt{\pi} \cdot \sqrt{\pi}$
- **D.** $2\sqrt{3} \sqrt{12}$
- **4.** Which number is *not* irrational?
 - **A.** $2 + \sqrt{3}$
- **c.** $\sqrt{4} + \sqrt{3}$
- B. $\sqrt{4} \sqrt{9}$ D. $\sqrt{4} \cdot \sqrt{3}$

- **5.** Which statement is *always* true?
 - A. The difference of two rational numbers is irrational.
 - **B.** The difference of two irrational numbers is irrational.
 - **C.** The product of an irrational number and a nonzero rational number is irrational.
 - **D.** The product of two irrational numbers is irrational.

- **6.** Which statement is *never* true?
 - A. The sum of a rational number and an irrational number is rational.
 - **B.** The sum of two irrational numbers is rational.
 - C. The product of two irrational numbers is rational.
 - **D.** The product of a nonzero rational number and an irrational number is rational.

7. Identify the rational numbers.

Select all the numbers that apply.

- a. $-2(12.\overline{7})$
- **b.** $14.8 + \frac{4}{9}$
- **c.** $\pi\sqrt{81}$
- **d.** $-\left(\sqrt{\pi^2}\right)$
- e. $\frac{9}{10} \frac{1}{2}$
- f. $\sqrt{225} \sqrt{121}$

- 8. Identify the irrational numbers. Select all the numbers that apply.
 - **a.** $1\frac{9}{16} + 0.75$
 - **b.** $-45.\overline{07} + \sqrt{19}$
 - **c.** $3\pi + \frac{2}{5}$
 - **d.** $0.\overline{157} + \frac{7}{12}$
 - e. $\frac{\sqrt{3}}{15}$

9. The table shows properties of real numbers. Identify each property as true or false.

	True	False
The sum and difference of two rational numbers are rational numbers.		
The product of two rational numbers is an irrational number.		
The sum of an irrational number and a rational number is a rational number.		
The product of an irrational number and a nonzero rational number is an irrational number.		
The set of irrational numbers are closed under addition.		

Use It On the Job

10. A ship captain can estimate the distance $d \mid 11$. Dyani is an architect in charge of building in miles to the horizon over water using the formula $d = 1.2246\sqrt{h}$, where h is the height of the captain's eyes above sea level.



For which value of h will d be a rational number?

- A. $\sqrt{4}$
- **B**. 9
- c. $\sqrt{9}$
- **D.** 14

a sloped roof. She knows the height h of the roof and the length ℓ for it to cover. To find the diagonal length d of the roof, she uses the equation slope equation $d = \sqrt{h^2 + \ell^2}$.



For which values of *h* and ℓ will *d* be an irrational number?

A.
$$h = 3$$
, $\ell = 4$

B.
$$h = 8$$
, $\ell = 6$

C.
$$h = 12, \ell = 5$$

D.
$$h = 15$$
, $\ell = 9$