



Geometry

APPLICATION LESSONS

STUDENT
EDITION





Career-Connected Learning

First, let us reflect on the words of renowned American educational reformer and philosopher John Dewey, “To find out what one is fitted to do and to secure an opportunity to do it is the key to happiness (1916/1999, p. 308).” With this guidance in mind, the **Pathway2Careers™ Curriculum helps *all* students to attain a successful future through career exploration and mathematics.**

Pathway2Careers (P2C) Curriculum was developed to align with Career-Connected Learning (CCL) principles to support sustainable career readiness. CCL, introduced by the National Center for College & Career Transitions, exposes students to the world of work during the learning process (Meeder & Pawlowski, 2020).

P2C serves as a bridge bringing together career-connected learning strategies and academic content. It engages students in mathematical reasoning using examples from specific industries, such as construction and healthcare. It also teaches essential transferable work skills such as teamwork, critical thinking, problem-solving, creativity, and communication.

These applications aim to engage in students' learning processes deeply. Students develop a more substantial interest in the content by connecting teaching and learning to the real world. In addition, each learner examines college and career opportunities that align with their strengths and values (Kemple & Willner, 2008). According to scholars Meeder and Pawlowski, students with greater exposure to career-connected learning, specifically career and technical education (CTE), are more likely to graduate from high school, enroll in a two-year college, be employed, and earn higher wages (2020). Bierly & Smith (2022) also reference the significant contributions of early college and career-connected learning models.

The Pathway2Careers Curriculum improves the flexibility and accessibility of CTE and engages more students in fully realized career pathways. P2C provides curricula that support career readiness for all students. Ultimately, career-connected learning helps students understand the “why” behind the “what.” As cited in several studies, when students are provided with tools to make learning relevant and applicable to themselves, their motivation, performance, retention, and interest increase (Malka & Covington, 2005; Jang, 2008; González et al., 2009; Marzano & Pickering, 2011; Yeager & Dweck, 2012; Hulleman et al., 2014).



Math Application Lessons

About

Every lesson in the Pathway2Careers (P2C) Math Curriculum directly connects to occupations that use the skills being taught. P2C is comprised of Math Application and Math Exploration lessons:

In Math Exploration lessons, students learn foundational math skills and concepts while exploring multiple careers and how the math skills connect to those careers. These skills are carried forward into the application lessons.

In the Math Application lessons, students work through real-world scenarios by applying math within the context of a single career. These lessons give students a deeper understanding of the connection between the occupation and the concept.

Math Application Lesson Structure

Career Spotlight

Career Spotlight sections are found on the first page of every Math Application lesson. It includes a description of the occupation, the educational requirements for the occupation, potential employers that hire people within the occupation, a short video of someone in the occupation discussing their work, salary and job openings projections, responsibilities, and more.

Lesson Objective

Learning goals and objectives are clearly established in every lesson in the “Lesson Objective” section. Students and teachers view this before beginning the lesson, understanding exactly what they should be able to do upon completion of the lesson.

Math Essentials

The “Math Essentials” sections introduce students to the topics of the lesson including vocabulary, prior knowledge, skills, etc.



Step Into the Career

In the “Step Into the Career” sections, students are presented with a problem that an employee in the spotlighted career may face. Here, it is encouraged that teachers have students consider how one might approach such a problem before continuing to the solution.

Once students have considered the problem, a plan is created in the “Devise a Plan” section and then executed in the “Walk Through the Solution” section. Here students may ask questions and inquire about the plan, the process, or the solution.

On the Job

“On the Job” sections are ideal for encouraging inquiry-based thinking and collaborative learning. They are designed for teachers to help students practice solving similar problems on their own or in small groups. Completing these problems with teacher and group availability prepares students for the individualized practice to come.

Career Spotlight: Practice

Both the “Career Spotlight: Practice” and the “Career Spotlight: Check” sections provide real-world, age-appropriate application problems, similar to the ones presented throughout the lesson. Questions in these sections require students to apply what they learned about the mathematical concept presented and about the occupation that was highlighted. Upon completion of the lesson, students will know more about both the occupation and the mathematical concept. A review of student’s understanding of these problems allows teachers the opportunity to reteach the material to those students who do not understand the content.

Career Spotlight: Check

Both the “Career Spotlight: Practice” and the “Career Spotlight: Check” sections provide real-world, age-appropriate application problems, similar to the ones presented throughout the lesson. Questions in these sections require students to apply what they learned about the mathematical concept presented and about the occupation that was highlighted. Upon completion of the lesson, students will know more about both the occupation and the mathematical concept. A review of student’s understanding of these problems allows teachers the opportunity to reteach the material to those students who do not understand the content.

Geometry Table of Contents

This bundle includes Math Application Lessons only. Visit www.ns4ed.com to explore more of what Pathway2Careers has to offer.

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PROJECT	Expressing Geometry Properties with Equations	Health Informatics Specialists
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PROJECT	Congruence	Computer Network Architects
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PROJECT	Circles	Telecommunications Engineering Specialists
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Multiple careers

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Gaming Managers

PROJECT

Using Probability to Make Decisions

Computer Programmers

LESSON 1.5

Solve Problems Using Pairs of Angles



CAREER SPOTLIGHT: Occupational Therapist

Occupation Description

Occupational therapists treat injured, ill, or disabled patients through the therapeutic use of everyday activities. They help these patients develop, recover, improve, and maintain the skills needed for daily living and working.

In some cases, occupational therapists help patients create functional work environments. They evaluate the workspace, recommend modifications, and meet with the patient's employer to collaborate on changes to the patient's work environment or schedule.

Education

Occupational therapists need at least a master's degree in occupational therapy; some therapists have a doctoral degree. Occupational therapists also must be licensed.

Potential Employers

The largest employers of occupational therapists are as follows:

Hospitals; state, local, and private	27%
Offices of physical, occupational, and speech therapists, and audiologists	26%
Elementary and secondary schools; state, local, and private	11%
Home healthcare services	9%
Nursing care facilities (skilled nursing facilities)	9%

Watch a video about occupational therapists:

<https://cdn.careeronestop.org/OccVids/OccupationVideos/29-1122.00.mp4>

Career Cluster

Health Science

Career Pathway

Therapeutic Services

Career Outlook

- Salary Projections:
Low-End Salary, \$56,800
Median Salary, \$84,950
High-End Salary, \$121,490
- Jobs in 2018: 133,000
- Job Projections for 2028:
156,800 (increase of 18%)

Geometry Concepts

- Apply the Angle Addition Postulate.
- Apply angle bisectors and find complementary and supplementary angles.

Is this a good career for me?

Occupational therapists:

- Evaluate a patient's condition and needs.
- Develop a treatment plan for patients.
- Help people with various disabilities perform different everyday tasks.
- Educate a patient's family and employer about how to accommodate and care for the patient.

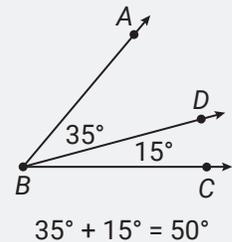
Lesson Objective

In this lesson, you will look at how an occupational therapist uses the concepts of angles and angle measurement when working to help patients regain full motion of their arms and legs.

Angle Addition Postulate

If D is in the interior of $\angle ABC$, then

$$m\angle ABD + m\angle DBC = m\angle ABC.$$



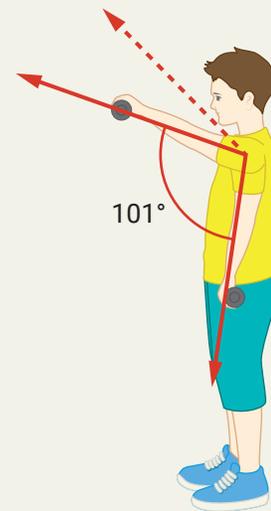
1 Step Into the Career: Angle Measurement

An occupational therapist is working with a patient who has a current range of motion for swinging his right arm up at the shoulder at the angle measure shown in the diagram.

The therapist has a goal of improving the patient's range by 26° . What is the goal for the total angle measure for the patient's range of motion?



An occupational therapist uses a goniometer to measure angles. It is similar to a protractor.



Devise a Plan

Step 1: Determine the angle measure for the current range of motion.

Step 2: Apply the Angle Addition Postulate and write an expression representing the total number of degrees for the goal range of motion.

Step 3: Find the value of the expression to determine the total number of degrees.

Walk Through the Solution

Step 1: Determine the current range of motion.

The diagram shows that the current range of motion is 101° .

Step 2: Applying the Angle Addition Postulate, the goal for the total range of motion can be found by using the expression $101^\circ + 26^\circ$.

Step 3: Find the value of the expression.

$$101^\circ + 26^\circ = 127^\circ$$

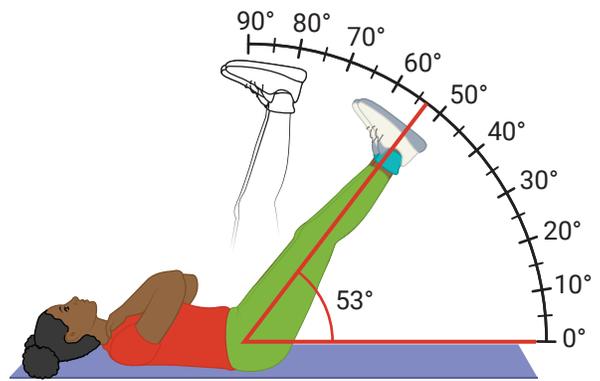
The goal for the total range of motion is 127° .

On the Job: Apply Angle Measurement

1. Before starting therapy, a patient had a range of motion in her legs of 53° .

After several weeks of therapy, the occupational therapist recorded an angle measure of 77° . The goal for complete recovery is a range of motion of 82° .

- a. What is the measure of the angle representing the improvement from 53° to 77° ?
- b. How many more degrees of motion are needed for complete recovery?
- c. What is the measure of the angle representing the improvement from 53° to complete recovery at 82° ?

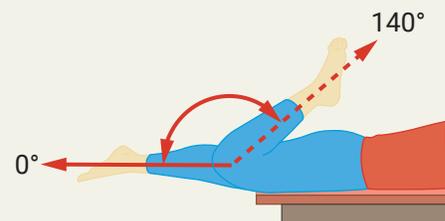


2 Step Into the Career: Angle Bisector

An occupational therapist is working with a patient who has had major knee surgery and currently has limited motion in her right knee.

The full range of motion for a healthy knee is 140° , as shown in the diagram.

The occupational therapist and patient decide that the first therapy target will be to gain back at least half of the full range of motion. How many degrees represents half of the range of motion?



Devise a Plan

An **angle bisector** divides an angle into two congruent angles. In this case, the occupational therapist is applying the angle bisector of the angle that represents the full range of motion to determine the first target.

Step 1: Determine the full range of motion for a healthy knee.

Step 2: Apply the concept of an angle bisector to write an expression representing the measure of each angle resulting from the angle bisector.

Step 3: Find the value of the expression to determine the number of degrees that represents half of the full range of motion.

Walk Through the Solution

Step 1: Determine the full range of motion.

The full range of motion for a healthy knee is 140° , according to the information given in the problem.

Step 2: The measure of each angle created by the angle bisector can be represented by the expression $\frac{140^\circ}{2}$.

Step 3: Find the value of the expression.

$$\frac{140^\circ}{2} = 70^\circ$$

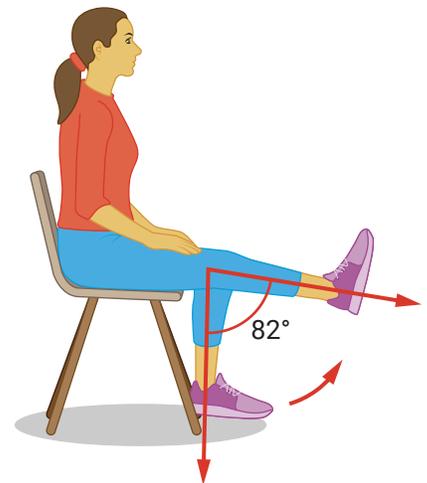
Half of the full range of motion is 70° .

On the Job: Apply Angle Bisector

2. An occupational therapist is working with a patient doing leg lifts on the right leg. A complete leg lift, bending at the knee, would have the total range of motion angle shown in the diagram.

The occupational therapist decides that a goal of half the total range of motion is a good first step.

- What is the angle measure for the total range of motion?
- What is the angle measure for the first step?

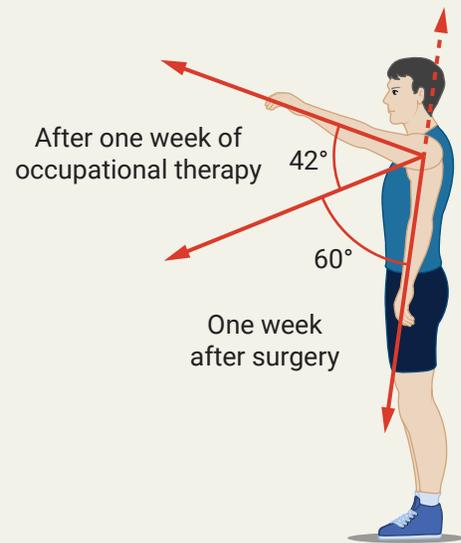


3 Step Into the Career: Supplementary or Complementary Angles

A professional baseball player is recovering from left shoulder surgery. The player begins occupational therapy one week after surgery. The occupational therapist's goal is for a return to a full shoulder extension range of motion of 180° , as shown by the dashed arrow.

What is the total range of motion accomplished after one week of occupational therapy?

How many more degrees does the player need to improve to attain the full range of 180° ?



Devise a Plan

Supplementary angles are two angles whose measures add to 180° . The goal for the total range total range of motion is 180° . So, the angle for the current amount of progress and the angle still needed for complete motion are supplementary angles.

Step 1: Add the range for one week after surgery to the additional range gained after one week of occupational therapy to find the angle of the range of motion for the current progress.

Step 2: Since you need to find the angle that is supplementary to the angle for the current progress, subtract the angle for the current progress from 180° .

Walk Through the Solution

Step 1: The range of motion one week after surgery is 60° . The additional range of motion gained after one week of occupational therapy is 42° . Add the angle measures.

$$60^\circ + 42^\circ = 102^\circ$$

Step 2: The full range of motion is 180° . Subtract 102° from 180° .

$$180^\circ - 102^\circ = 78^\circ$$

To attain full range of motion, the player needs to improve by 78 more degrees.

On the Job: Apply Supplementary or Complementary Angles

3. An occupational therapy patient is lying on a mat working on improving the movement in her right leg. The goal is to move the leg from flat on the floor to extended straight above her. This is a right angle. The current angle she can move her leg to is complementary to the angle she still needs to attain.



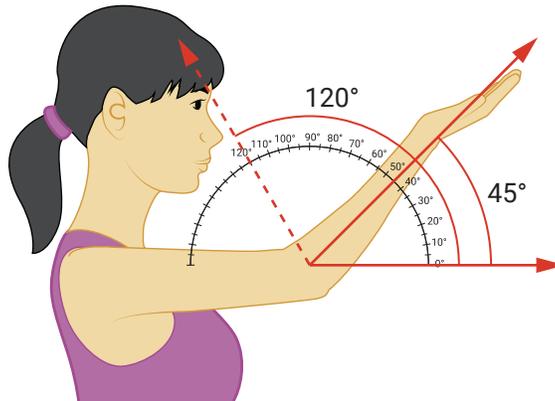
QUICK TIP

Complementary angles are two angles whose measures add to 90° .

If the patient can move her leg 28° , how many more degrees does she need to move to reach her goal?

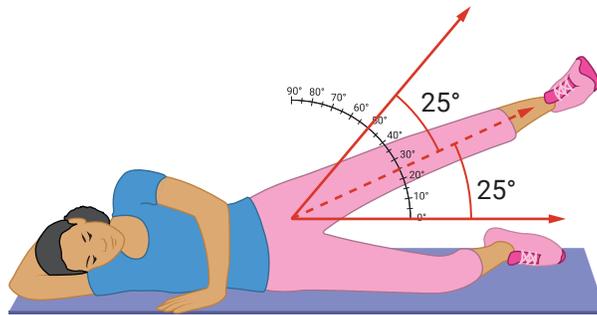
Career Spotlight: Practice

4. After elbow surgery, a patient's range of motion angle for elbow flexion is shown on the goniometer. The dashed arrow indicates the goal range of motion.

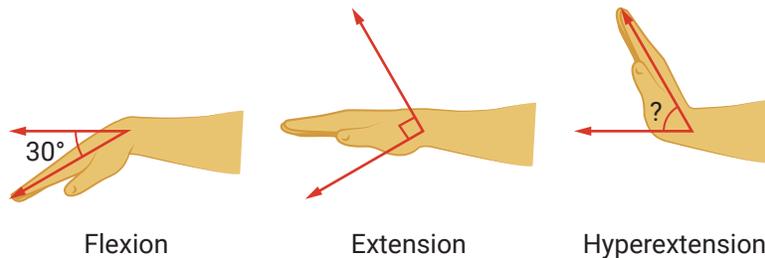


- What is the angle measure for the current range of motion for the patient?
- What is the angle measure for the goal range of motion?
- What is the difference in degrees between the current range of motion and the goal range of motion?

5. Hip abduction is the sideways motion of the leg from the hip, as shown in the picture. The typical full range of motion angle for hip abduction is indicated on the goniometer by the solid arrows. The leg of the person in the picture forms an angle bisector to the angle for the full range of motion, as shown by the dashed arrow.



- What is the angle measure for the typical full range of motion?
 - What number of degrees represents half of the typical range of motion?
 - How many more degrees of motion are necessary for the person to attain the typical range of motion?
6. An occupational therapist uses a goniometer and finds that the right wrist of a patient has a total range of motion from flexion to hyperextension that is a right angle. The range of motion angle from flexion to extension is 30° . What is the measure of the range of motion angle from extension to hyperextension? (**HINT:** The angle is complementary to the angle from flexion to extension.)



Devise a Plan

Step 1: Determine the known angle measures.

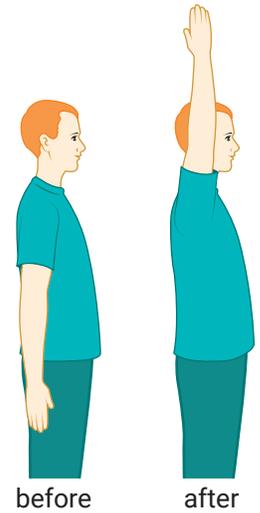
Step 2: _____ ? _____.

Step 3: _____ ? _____.

7. An occupational therapist is working with a patient on front shoulder extensions for the right arm. This movement begins with the arm hanging loosely at the side. The person keeps the arm straight and lifts the arm in front of the body, finishing with the arm straight upward, as shown in the diagram.

The patient has a 105° range of motion currently. The additional angle the patient needs to be able to move to attain full range of motion is supplementary to the current angle.

What is the measure of the remaining angle?



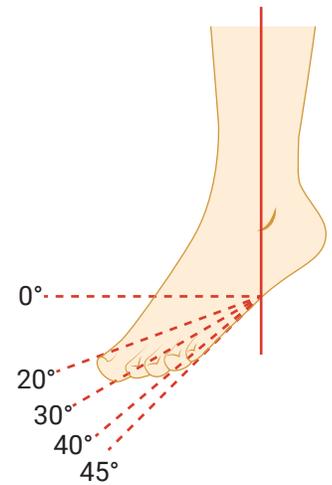
Career Spotlight: Check

8. The diagram shows the typical range of motion for a plantar flexion of the ankle.

A gymnast is recovering from ankle surgery and currently has a plantar flexion range of 22° .

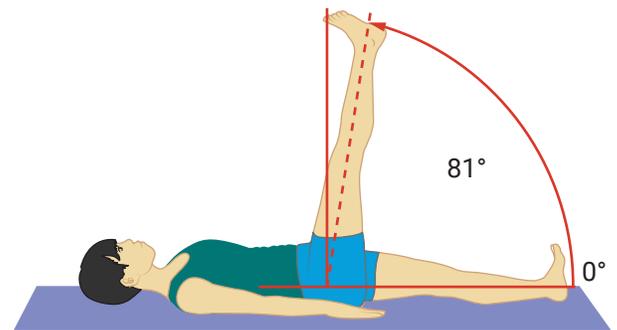
By how many more degrees does the gymnast need to improve to show full range of motion for a plantar flexion?

- A. 22°
- B. 23°
- C. 45°
- D. 78°

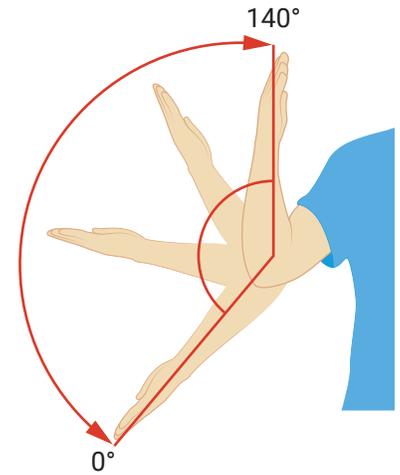


9. An occupational therapist is working with a patient who is recovering from hip surgery.

For the first week of therapy, the goal for the patient's initial range of motion is half of the patient's normal range of motion, which is shown in the diagram. What is the angle measure of the initial goal?



10. An occupational therapist is working with a patient who had elbow surgery. The full range of motion for the elbow is 140° , as shown in the diagram. The therapist wants to use 25% of the total range of motion for the first goal and 50% of the total range of motion for the second goal of the therapy. Note that 50% of the full range bisects the full range angle and 25% bisects the 50% angle.



25% of the full range is

- | |
|---------------|
| a. 25° |
| b. 50° |
| c. 35° |

50% of the full range is

- | |
|----------------|
| a. 50° |
| b. 70° |
| c. 140° |

After attaining 50% of full range, the patient will need another

- | |
|---------------|
| a. 25° |
| b. 50° |
| c. 70° |

to achieve full range of motion.

11. An occupational therapist finds that a patient has a range of motion of 36° . This is 60% of the typical full range of motion. Which is the typical full range of motion?
- A. 18°
 - B. 36°
 - C. 45°
 - D. 60°

12. An occupational therapist is creating a help sheet for different ranges of motion. She wants to have a quick reference for angles that are 40% of the typical total range of motion angles. Match each total range of motion angle measure with the angle measure representing 40% of the motion.

	36°	48°	72°
Wrist flexion to hyperextension, 90°	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Elbow flexion, 120°	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Shoulder neutral to flexion, 180°	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

13. The diagram shows range of motion for a vertical abduction of the shoulder.

Select all the statements that are true.

- The angle formed by the angle bisector of the total range of motion is 90°.
- The total range of motion from neutral to abduction is 360°.
- A patient who has a range of 135° of motion for vertical shoulder abduction has 75% of the normal range of motion.
- One-third of the normal range of motion is about 45°.
- Half of the normal range of motion is about 80°.

